Elasticity of Demand

Q : If we increase the price of a product by a certain percentage, what percentage change is there to the demand?

Example p = price per unit

q = number of units sold

Demand Equation : $q = 100 - 2p$

Notice we have written this with p as independent variable.

a) If $p = 10$ and we increase the price by 10% what is the percentage change to q?

$p = 10 \Rightarrow q = 100 - 2 \times 10 = 80$

10% increase to $p \Rightarrow p = 11 \Rightarrow q = 100 - 2 \times 11$
$= 78$

$\Rightarrow$ Net change to $q = 78 - 80 = -2$

$\Rightarrow$ Percentage change to $q = \dfrac{-2}{80} \times 100 = -2.5\%$

Conclusion : When $p = 10$, $q = 80$, a 10% increase in price results in a 2.5% decrease in demand.

b) If $p = 30$ and we increase the price by 10% what is the percentage change to q?

$p = 30 \Rightarrow q = 100 - 2 \times 30 = 40$

10% increase to $p \Rightarrow p = 33 \Rightarrow q = 100 - 2 \times 33 = 34$

$\Rightarrow$ Net change to $q = 34 - 40 = -6$

$\Rightarrow$ Percentage change to $q = \dfrac{-6}{40} \times 100 \approx -15\%$

$\boxed{\text{\underline{Conclusion}: When } p = 30,\ q = 40, \text{ a } 10\% \text{ increase in price results in a } 15\% \text{ decrease in demand.}}$

<u>Remark</u> Looking at this example we see that the ratio:

$$\frac{\text{Percentage change in } q}{\text{Percentage change in } p}$$

is different depending on the starting p.

General Picture :

p = price per unit

q = number of units sold.

Demand Equation : $q = D(p)$

p is independent variable

h = Net change in p

$\Rightarrow$ Net change in $q = D(p+h) - D(p)$

$\Rightarrow$ Percentage change in price $= \dfrac{h}{p} \times 100\ \%$

Percentage change in demand $= \dfrac{D(p+h) - D(p)}{D(p)} \times 100\ \%$

$\Rightarrow$

$$\frac{\text{Percentage change in demand}}{\text{Percentage change in price}} = \frac{\left(\frac{D(p+h)-D(p)}{D(p)}\right) \times 100}{\left(\frac{h}{p}\right) \times 100}$$

$$= \frac{p}{D(p)} \cdot \underbrace{\frac{D(p+h)-D(p)}{h}}_{\text{difference quotient}}$$

Remark : If $h > 0$ the demand will decrease so this ratio will be negative.

Definition (Elasticity of Demand)

$$\underset{\text{"elasticity at price } p\text{"}}{E(p)} = \lim_{h \to 0} \left(- \frac{\text{Percentage change of demand}}{\text{Percentage change of price}} \right)$$

$$= \lim_{h \to 0} \frac{-p}{D(p)} \cdot \frac{D(p+h) - D(p)}{h}$$

$$= \frac{-p}{D(p)} \cdot D'(p)$$

Will be positive as $D(p)$ decreasing function.

$$q = D(p) \Rightarrow E(p) = \frac{-p}{q} \cdot \frac{dq}{dp}.$$

Intuition : If the price p is increased by $A\%$ (and A is small), then the demand q will decrease by $E(p)\,A\%$.

Remark :

Demand is inelastic if $E(p) < 1$

Demand is elastic if $E(p) > 1$

Demand has unit elasticity if $E(p) = 1$

The demand for highly inelastic products is less responsive to changes in price (e.g. Medical services)

The demand for highly elastic products is more responsive to changes in price (e.g. restaurant meals)

Example Assume $q = 216 - 2p^2$. Find the price intervals where the demand is elastic / inelastic.

$$q, p > 0 \Rightarrow 216 - 2p^2 > 0 \Rightarrow \frac{216}{2} > p^2$$

$$\Rightarrow p \text{ in } (0, \sqrt{108})$$

$$\frac{dq}{dp} = -4p$$

$$E(p) = \frac{-p}{q} \cdot \frac{dq}{dp} = \frac{-p}{216 - 2p^2} \cdot (-4p) = \frac{4p^2}{216 - 2p^2}$$

We need to do sign analysis on $E(p) - 1$.

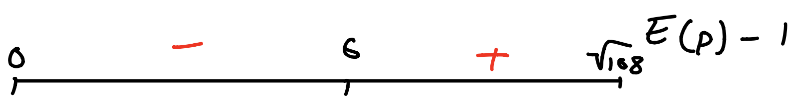
$$E(p) - 1 = \frac{4p^2}{216 - 2p^2} - 1 = \frac{4p^2 - (216 - 2p^2)}{216 - 2p^2}$$

$$= \frac{6p^2 - 216}{216 - 2p^2}$$

A/ $E(p) - 1 = 0 \Rightarrow 6p^2 - 216 = 0 \Rightarrow p^2 = 36$

$\Rightarrow p = \pm 6$

B/ $E(p) - 1$ continuous everywhere on $(0, \sqrt{108})$.



$E(1) - 1 < 0$ $\quad\quad$ $E(7) - 1 > 0$

$E(p) - 1 < 0 \Rightarrow E(p) < 1$

$E(p) - 1 > 0 \Rightarrow E(p) > 1$

$\Rightarrow$ $E(p)$ inelastic on $(0, 6)$

$E(p)$ elastic on $(6, \sqrt{108})$

$E(p)$ unit elasticity when $p = 6$

Elasticity and Revenue

revenue as a function in p.

$R(p) = pq = p\,D(p)$ (Always assume $p, q > 0$)

$$\Rightarrow \frac{dR}{dp} = \frac{d}{dp}(p) \cdot D(p) + p \cdot \frac{d}{dp}(D(p))$$

$$= D(p) + p\,D'(p)$$

$$\Rightarrow \quad \frac{dR}{dp} = D(p)\left(1 + \frac{p}{D(p)} D'(p)\right)$$

$$= D(p)\left(1 - \left(\frac{-p}{D(p)} D'(p)\right)\right)$$

$$= D(p)\left(1 - E(p)\right)$$

$$\Rightarrow \quad \frac{dR}{dp} > 0 \quad \text{iff} \quad 1 - E(p) > 0$$

$$(\text{ie } E(p) < 1)$$

inelastic

$$\frac{dR}{dp} < 0 \quad \text{iff} \quad 1 - E(p) < 0$$

$$(\text{ie } E(p) > 1)$$

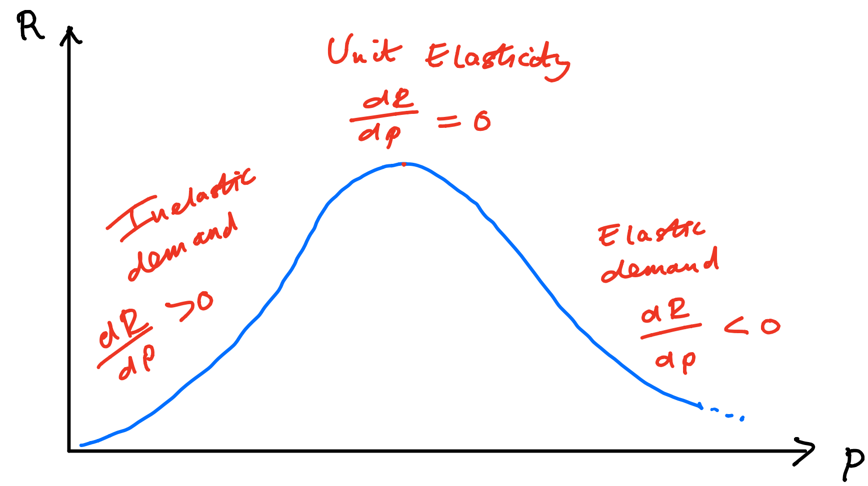
elastic

$$\frac{dR}{dp} = 0 \quad \text{iff} \quad 1 - E(p) = 0$$

$$(\text{ie } E(p) = 1)$$

unit elasticity

Picture:



Conclusion

1/ If demand is inelastic, revenue increases as price increases

2/ If demand is elastic, revenue decreases as price increases.

3/ Total revenue is maximized at the price where demand has unit elasticity.

Example

If a product is being sold at a price where the elasticity is 1.5, should the company increase or decrease the price for higher revenue?

$E(p) = 1.5 \Rightarrow$ elastic at p $\Rightarrow$ R decreasing at p

$\Rightarrow$ Company should decrease the price.